Scaling of the thermoelectric power in the $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}\ (0 \leqslant x \leqslant 0.7)$ and

$Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}\ (0 \leqslant x \leqslant 0.6)$ systems

Scaling of the thermoelectric power in the Bi$_2$Sr$_2$Ca$_{1-x}$Y$_x$Cu$_2$O$_{8+y}$ (0 ⩽ x ⩽ 0.7) and Tl$_2$Ba$_2$Ca$_{1-x}$Y$_x$Cu$_2$O$_{8+y}$ (0 ⩽ x ⩽ 0.6) systems

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Abstract. The temperature-dependent thermoelectric power S of insulating and metallic Bi-2212 and Tl-2212 samples are scaled to universal master curves. For insulating samples, S passes through a peak at a characteristic temperature T_S^*, which decreases with increasing carrier concentration. For metallic samples a scaling parameter α (= dS/dT at high temperatures) decreases with increasing carrier density. It is argued that T_S^* is related to the energy needed for activation conduction whereas the change in α may be due to increase in n/m^* with carrier density, where n is the concentration and m^* is the effective mass of the carriers.

The thermoelectric power (TEP) is one of the transport properties, complementary to the electrical resistivity and the Hall coefficient, measured by various groups for different high-T_c systems [1–3]. The TEPs S of high-T_c cuprates have common characteristics. For semiconducting or insulating samples with a low carrier concentration, S is large and positive and its value decreases with increase in the carrier concentration. For metallic samples, S is small and shows an almost linear temperature dependence at high temperatures with a negative slope. For overdoped samples with high carrier densities, S is negative.

Different groups [4–7] have demonstrated that the various normal-state transport properties of high-T_c superconducting samples can be scaled to universal master curves. Ito *et al* [4] have shown that the in-plane resistivity of YBa$_2$Cu$_3$O$_{7-x}$ deviates from linear behaviour below a temperature T_ρ^* well above T_c. T_ρ^* decreases as the oxygen concentration (or the carrier concentration) increases. They could scale the resistivity of all the samples in the doping range $x = 0.10$–0.55. Hwang *et al* [5] found that the temperature-dependent Hall coefficient $R_H(T)$ in La$_{2-x}$Sr$_x$CuO$_4$ ($x = 0.15$–0.34) can be scaled to a universal functional form

$$R_H(T) = R_H^\infty + R_H^* f\left(\frac{T}{T_H^*}\right). \tag{1}$$

$R_H(T)$ is temperature dependent in the low-temperature region and becomes temperature independent (R_H^∞) when T exceeds a certain characteristic value T_H^*. R_H^* rescales the magnitude of $f(T/T_H^*)$. The characteristic temperatures T_χ^* and T_K^* may be defined from the susceptibility [8] and the Knight shift [5, 9] data where the susceptibility or the Knight shift shows a peak. It is found that T_H^*, T_χ^* and T_K^* are close to each other for a fixed doping concentration and decrease with increasing carrier concentration [5].

From the study of the resistivity and Hall coefficient, Chen *et al* [6] also found that the temperature dependence of the Hall coefficient in the 123 system can be scaled to the same functional form (1). They also observed that the characteristic temperature T_ρ^*, below which $\rho(T)$ deviates from the linear behaviour, and T_H^* are close to each other and decrease with increasing carrier concentration.
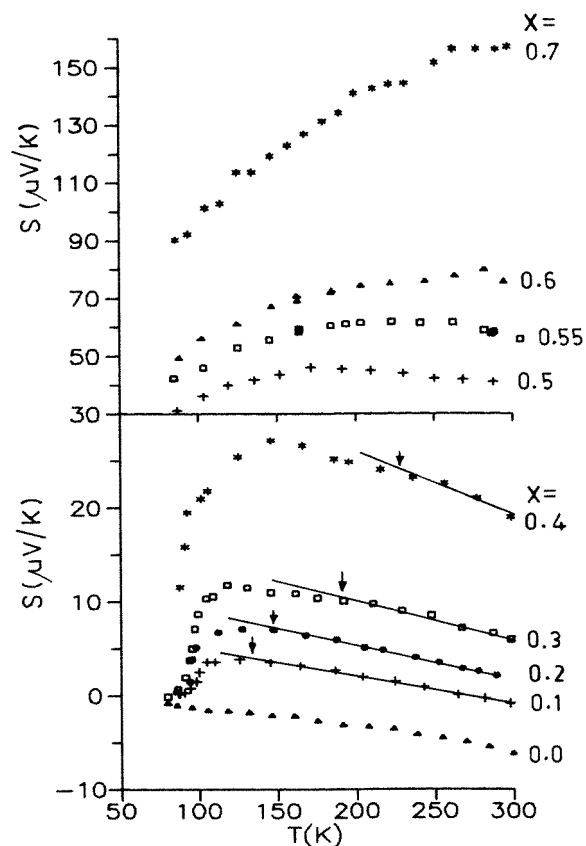

Figure 1. Temperature dependence of the TEP for $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ ($0 \leqslant x \leqslant 0.7$). For metallic samples the T_S^0-values are indicated by arrows.

So far scaling was done on materials with $T_c > 0$ and the properties considered are the resistivity, Hall constant, Knight shift and susceptibility. No one has tested the scaling behaviour on non-superconducting samples and moreover the scaling of the TEP results in high-T_c systems has not been addressed so far. In this communication we have analysed the TEP data of Bi-2212 and Tl-2212 systems and found that the normal-state TEP of non-superconducting as well as superconducting samples can be scaled to universal master curves.

In earlier communications we reported the temperature-dependent TEP [3] of the $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ (Bi-2212) and $Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}$ (Tl-2212) systems. Our results are in good agreement with other published data on polycrystalline and single-crystal samples [2]. In the present paper we use the results of [3] for analysis. The
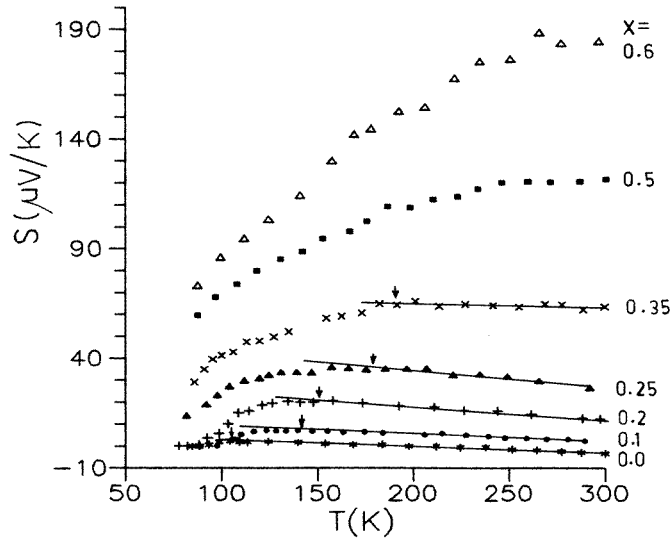
Figure 2. Temperature dependence of the TEP for $Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}$ ($0 \leqslant x \leqslant 0.6$). For metallic samples the T_S^0-values indicated by arrows.
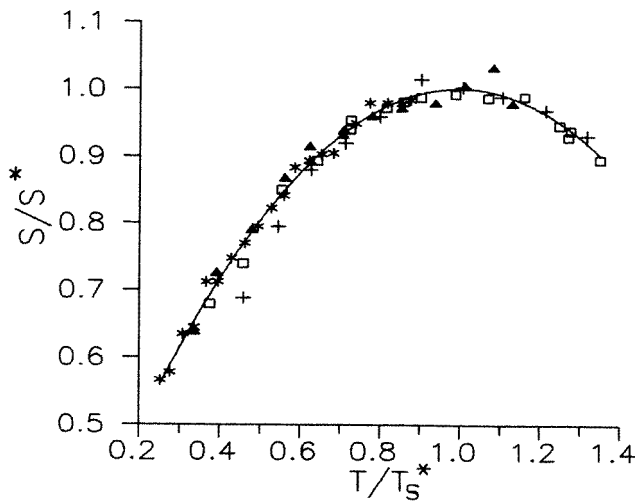
Figure 3. S/S^* versus T/T_S^* for $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ samples with $x = 0.5$ (+), 0.55 ($\square$), 0.6 ($\blacktriangle$) and 0.7 (*).

temperature and carrier dependences of S for $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ ($0 \leqslant x \leqslant 0.7$) and $Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}$ ($0 \leqslant x \leqslant 0.6$) systems [3] are shown in figures 1 and 2, respectively. The carrier concentration in these systems decreases with increasing x. For Bi-2212 a metal-to-insulator (M–I) transition occurs at $x = x_c \simeq 0.50$, whereas for the Tl-2212 system a similar M–I transition occurs at $x_c \simeq 0.48$ [10]. Superconductivity appears in these systems for $x < x_c$ (i.e. in the metallic region). Samples for which $d\rho/dT$

is positive and ρ is linear in T at high temperatures are referred to as metallic samples whereas insulating samples are characterized by a negative $d\rho/dT$ and an exponential rise in ρ with decreasing temperature [10].
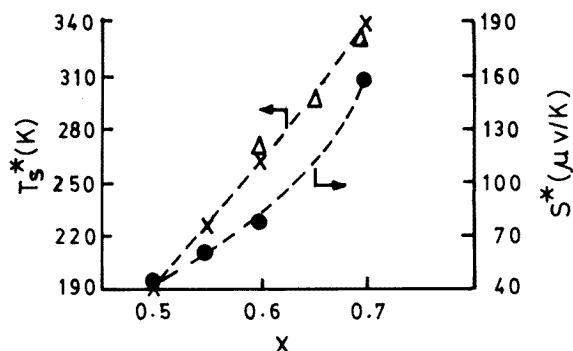

Figure 4. Variations in T_S^* and S^* with x for $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ samples. Variation in $(E_c - E_F)/k_B$ ($\triangle$) with x is also shown (data taken from [10, 12]). To bring the data points within the left-hand scale, 150 K is added to $(E_c - E_F)/k_B$.

For insulating samples the TEP shows a broad peak at a characteristic temperature T_S^* which decreases with decreasing x (figures 1 and 2). We find that, on rescaling T by T_S^* and S by S^* (S^* is the TEP value at $T = T_S^*$), the TEP data of insulating samples can be fitted to a single master curve. The scaled TEP data of $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ ($x = 0.50$, 0.55, 0.6 and 0.7) samples are shown in figure 3. For the $x = 0.7$ sample, T_S^* and S^* are determined by extrapolating the TEP data to high temperatures. For the $x = 0.5$ sample the data points deviate from the universal curve at low temperatures. The carrier concentration of this sample lies close to the crossover region of the insulator-to-metal transition [10]. We find that the data for samples in the crossover region do not fit with the scaling of either the insulating or the metallic samples.

In figure 4 we have shown the variation in T_S^* and S^* with x and found that both S^* and T_S^* increase with increasing x, i.e. with decreasing carrier concentration.

For metallic Bi samples at high temperatures ($T > T_S^0$) the TEP follows a linear temperature dependence with negative slope: $S = S_0 - \alpha T$. The temperature T_S^0, below which the deviation of S from the linearity occurs, shifts to higher values as the carrier concentration decreases (x increases). In figure 5(a) the variation in T_S^0 with x is shown. It increases as x increases. The T_S^0-values for the Bi-2212 single crystals (estimated from the data of Mandrus *et al* [2(a)]) are also shown in the same figure. For both polycrystalline and single-crystal samples the T_S^0-values show similar variations with x, indicating that this behaviour is intrinsic. In figure 6 we have plotted the variation in $(S - S_0)/\alpha$ versus T for $x = 0$, 0.1, 0.2, 0.3 and 0.4. All the data points fit a universal curve quite well in the high-temperature region. Deviation from the universal curve occurs at a temperature which decreases with increasing carrier concentration of the samples (as expected from the behaviour of T_S^0).

In figure 5(b) the variations in S_0 and α with x are shown. It is found that S_0 is higher for low-carrier-concentration (high-x) samples and it decreases with increasing carrier concentration. α increases linearly with x. The variation in $d\rho/dT$, calculated from the high-temperature resistivity data for Bi-2212 [11(a)] with x is also shown in figure 5(b). It is seen that both $d\rho/dT$ and α vary with x in a similar fashion. In table 1 we have listed
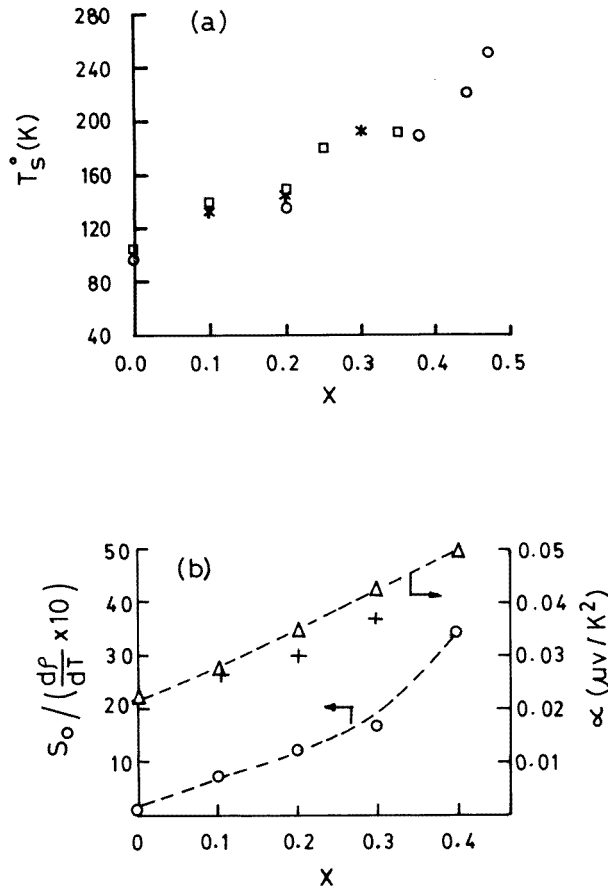
Figure 5. (*a*) Variation in T_S^0 with x: *, Bi-2212; □, Tl-2212; O , Bi-2212 single crystals (see text). (*b*) Variation in S_0 (μV K^{-1}) (O), α (△) and dρ/dT(mΩ cm K^{-1}) (+) with x for Bi$_2$Sr$_2$Ca$_{1-x}$Y$_x$Cu$_2$O$_{8+y}$.

the values of ρ_{300} (the resistivity at 300 K), T_c, dρ/dT (taken from [10, 11] S_{300} (the TEP at 300 K), S_0, T_S^0 and α for metallic samples and S_{300}, S^*, and T_S^* and $E_c - E_F$ for insulating samples of the Bi system.

For insulating Tl samples ($x = 0.5$ and 0.6) we have determined T_S^* and S^* in the same way as in the Bi system. Figure 7 shows the plot of S/S^* versus T/T^*. The scaled data points of the two samples are close to each other. The values of T_S^* and S^* are obtained as 323 K and 285 K, and 188.6 μV K^{-1} and 121.8 μV K^{-1} for $x = 0.6$ and 0.5 samples, respectively. Both T_S^* and S^* decrease with increasing carrier concentration.

The TEP data for metallic Tl samples also show a linear temperature dependence: $S = S_0 - \alpha T$, for $T > T_S^0$. The variation in T_S^0 with x is shown in figure 5(*a*). In figure 8 we plot $(S - S_0)/\alpha$ versus T. It is found that the data of the metallic samples ($x = 0$–0.25) may be described by a master curve (the TEP data for the $x = 0.35$ sample do not fit any of the master curves well). Deviation of the data points from the master curve occurs at low temperatures, similar to that observed in the Bi-2212 system. The variations in α, S_0 and dρ/dT [11(*b*)] with x for Tl samples are shown in the inset to figure 8. S_0, α and

Table 1. Various parameters extracted from the resistivity and thermoelectric power results for the $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ system.

x	ρ_{300} (mΩ cm)	$d\rho/dT$ (mΩ cm K^{-1})	T_c (K)	S_{300} (μV K^{-1})	S_0 (μV K^{-1})	T_S^0 (K)	α (μV K^{-2})
0	1.02	—	65	−6	1.1	—	0.022
0.1	1.20	2.70	71	−0.8	7.7	133.9	0.028
0.2	1.40	2.99	81	2.1	12.3	145.6	0.035
0.3	1.70	3.68	78	6.0	16.8	192.6	0.043
0.4	10.0	—	46	19	34.8	—	0.050

	$E_c - E_F$ (K)	S_{300} (μV K^{-1})	S^* (μV K^{-1})	T_S^* (K)
0.5	—	41	45.5	190
0.55	—	55.8	62.3	226
0.6	121	75.9	77.5	261
0.65	150	—	—	—
0.7	181	157	159.5	340
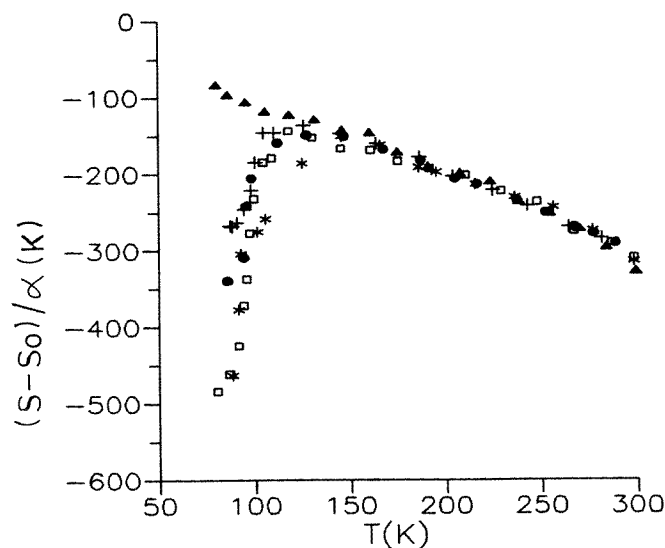
Figure 6. Plot of $(S - S_0)/\alpha$ versus T for $Bi_2Sr_2Ca_{1-x}Y_xCu_2O_{8+y}$ with $x = 0.0$ (▲), 0.1 (+), 0.2 (●), 0.3 (□) and 0.4 (*).

$d\rho/dT$ are large for high x, i.e. for low carrier concentrations, and decrease with increase in the carrier concentration. In table 2 we have presented the values of ρ_{300}, T_c, $d\rho/dT$ (taken from [10, 11]), and S_{300}, S_0, T_S^0 and α for metallic samples and S_{300}, S^* and T_S^* for insulating samples of the Tl system.

From the scaling of the TEP data for insulating samples of the Bi-2212 and Tl-2212 systems it is found that a characteristic temperature T_S^* exists, which decreases with increasing carrier concentration. The resistivity of these insulating samples ($x > x_c$) of the Bi and Tl systems shows variable-range hopping (VRH) transport in the low-temperature region and activated conduction in the high-temperature region [10, 12]. The resistivity
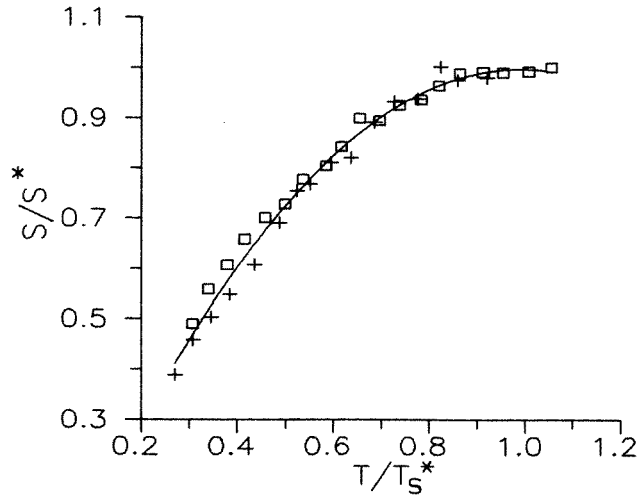
Figure 7. S/S^* versus T/T_S^* for $Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}$ with $x = 0.5$ (□) and 0.6 (+).
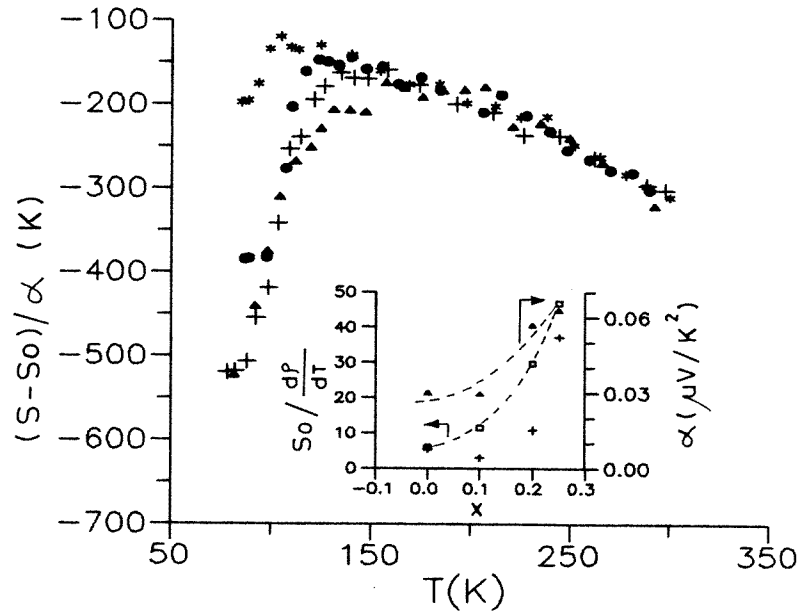
Figure 8. Plot of $(S - S_0)/\alpha$ versus T for $Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}$ with $x = 0.0$ (*), 0.1 (●), 0.2 (+) and 0.25 (▲). In the inset the variation in S_0 (μV K^{-1}) (□), α (▲) and $d\rho/dT$ ($m\Omega$ cm K^{-1}) (+) with x is shown.

results were analysed in terms of Anderson localization [12] within a Fermi glass model [13]. The presence of disorder in a periodic system causes localization of the electronic states in the tails of the conduction band; a mobility edge E_c separates the localized states from the extended states in the electronic band [13]. Zvyagin [14] studied the behaviour of

Table 2. Various parameters extracted from the resistivity and thermoelectric power results for the $Tl_2Ba_2Ca_{1-x}Y_xCu_2O_{8+y}$ system.

x	ρ_{300} (mΩ cm)	$d\rho/dT$ (mΩ cm K^{-1})	T_c (K)	S_{300} (μV K^{-1})	S_0 (μV K^{-1})	T_S^0 (K)	α (μV K^{-2})
0	2.58	5.55	103	−3.3	5.9	105	0.03
0.1	3.19	2.99	100	2.5	11.5	140	0.029
0.2	10.14	10.84	92	12.5	29.7	150.5	0.057
0.25	22.0	37.21	70	26.7	46.8	179.6	0.063
0.35				63.6		191.3	

x	S_{300} (μV K^{-1})	S^* (μV K^{-1})	T_S^* (K)
0.5	121.8	121.8	285
0.6	184.8	188.6	323

the TEP in the Fermi glass model and its dependence on T and $E_c - E_F$, where E_F is the Fermi energy. At low temperatures in the VRH region the TEP increases with increasing temperature as $T^{1/2}$. At higher temperatures, in the thermally activated conduction region, the TEP decreases with temperature and follows the relation [13, 14]

$$S = (k/e)(\epsilon/kT + \text{constant}) \tag{2}$$

where $\epsilon = E_c - E_F$. Thus in the Anderson localization picture the TEP passes through a peak as the conduction process changes from the VRH mechanism to thermally activated behaviour. The temperature T_S^* at which the maximum in S occurs in insulating Bi and Tl samples may then be interpreted as the temperature above which activated conduction becomes appreciable and should decrease with increasing $E_c - E_F$. The decrease in T_S^* with decreasing x (increasing carrier concentration) may be ascribed to the corresponding decrease in $E_c - E_F$ as x shifts towards x_c (at $x = x_c$, $E_c - E_F = 0$). Mandal and co-workers [10] and Quitmann *et al* [12] estimated $E_c - E_F$ from the analysis of the resistivity for the Bi system in the thermally activated region and their results agree well with each other to within 10%. It was observed that $E_c - E_F$ increases almost linearly with increasing x for $x < 0.9$. We find that T_S^* increases linearly with increasing x. This establishes a linear relation between T_S^* and $E_c - E_F$, which is demonstrated in figure 4.

From the scaling of the TEP data of the metallic Bi and Tl samples it is found that the coefficient α, which describes the slope of the S–T curve in the linear region, decreases with increasing carrier concentration. The $d\rho/dT$ values for the Bi-2212 and Tl-2212 systems also show similar variations with the carrier concentration. Ito *et al* [4] observed that $(d\rho/dT)^{-1}$ and the Drude spectral weight ω_{pD}^2 show similar doping dependences in the Y-123 system. Considering that

$$\rho = 4\pi/\omega_{pD}^2\tau \tag{3}$$

$\omega_{pD}^2 \propto n/m^*$ and $\tau^{-1} \propto T$ in the linear-T region of the resistivity, they argued that the change in $d\rho/dT$ for different samples is due to the change in m^*/n, where m^* and n are the effective mass and the concentration of the carriers, respectively, and τ is the relaxation time.

It may be mentioned that the TEP of a simple metal is given by [15]

$$S = -\frac{\pi^2 k_B^2 T}{2e\epsilon_F}. \tag{4}$$

The Fermi energy ϵ_F for a quasi 2D Fermi-liquid system is [16]

$$\epsilon_F = (\hbar^2 \pi d_c) \frac{n}{m^*} \tag{5}$$

where d_c is the interlayer distance. From equations (3)–(5), one obtains that both $\mathrm{d}\rho/\mathrm{d}T$ and $\mathrm{d}S/\mathrm{d}T$ are proportional to m^*/n. The similar doping dependences of α ($= \mathrm{d}S/\mathrm{d}T$) and $\mathrm{d}\rho/\mathrm{d}T$ in Bi and Tl samples, then, suggests that the variation in m^*/n with the carrier concentration may be responsible for the observed behaviour. It may be mentioned, however, that the simple formula for S should be taken with caution for metallic samples (even in the linear-T region) of high-T_c systems, which are not simple metals.

In high-T_c oxide systems it has been found from the plasma frequency [17] and the μSR measurements [18] that n/m^* increases with increasing carrier concentration. This behaviour is consistent with the decreases in α and $\mathrm{d}\rho/\mathrm{d}T$ with increasing carrier concentration.

For metallic samples of the Bi and Tl systems we observe that the TEP deviates from linear behaviour below a temperature T_S^0, which is lower for more metallic samples. As mentioned earlier, the resistivities of metallic samples of high-T_c systems show a deviation from linear behaviour below T_ρ^*. Both T_ρ^* and T_S^0 show similar variations with doping. The deviation of ρ from linearity below T_ρ^* is believed to be due to the formation of a spin gap in the spin excitation spectrum [4]. The gap develops well above T_c, as observed in inelastic neutron scattering [19] and NMR Knight shift measurements [20]. Whether the opening of a normal-state pseudo-gap in the charge and spin excitation spectrum is responsible for the deviation of S from linearity is an interesting problem for future study.

In conclusion we have found that the TEP of the Bi-2212 and Tl-2212 systems can be scaled to universal master curves for metallic as well as insulating samples. For insulating samples the characteristic temperature T_S^* decreases with increasing carrier concentration. T_S^* increases linearly with increasing $E_c - E_F$, determined from the resistivity analysis. For metallic samples a characteristic temperature T_S^0 also exists, which decreases with increasing carrier density. The decrease in the scaling factor α with increasing carrier concentration may be attributed to the increase in n/m^* with increasing carrier density.

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